**Problem 7.9.1**

**KNOWN:**
Freezing point of a solution is -6°C

**FIND:**
What is the solute concentration

**SCHEMATIC AND GIVEN DATA:**
Molecular weight of solution = 183.61
Amount of water and solute before freezing (85% water, 15% solute)

**STRATEGY:**
At the given temperature of -6°C for the entire solution, enough water would have frozen such that the freezing point of the solution is -6°C. Since the amount of solute has stayed the same as the initial amount, the solvent (water) amount has reduced (frozen water is no longer available) so that the solution is concentrated. From the solvent concentration at -6°C, obtained from the freezing point depression formulas, we can find the solvent amount at -6°C. By subtracting from the initial solvent amount, we can find the amount of the solvent (water) that must have changed to ice.

**ASSUMPTIONS:**

**SOLUTION:**
Problem 7.9.1

1) \[ \frac{\Delta H_f}{R} \left[ \frac{1}{T_{A_0}} - \frac{1}{T_A} \right] = \ln x_A \]

\[ \frac{6013.4 \text{ J/mol}}{8.315 \text{ J/mol K}} \left[ \frac{1}{273} - \frac{1}{267} \right] \]

\[ = \ln \frac{x/18}{.15 + x/18} \]

\[ \frac{x}{.15 + x/18} = .94221 \]

\[ .00321 x = \frac{(.94221)(.15)}{183.61} \]

\[ x = .2397 \]

Initially \( x = .85 \)

Fraction frozen = \[ .85 - .2398 \]

\[ .85 = .718 \]

\% Frozen = 71.8

2) Similarly for temperature of -15°C. \% Frozen = 89.6

3) The answer will be different for a more concentrated solution, because depression in freezing point is directly proportional to concentration.
Problem 7.9.7

**KNOWN:**
Air flow over a flat surface; thickness of a slab.

**FIND:**
Heat transfer coefficient over the surface; time for the slab to freeze.

**SCHEMATIC AND GIVEN DATA**

\[
T_\infty = -50^\circ C \\
u_\infty = 0.5 \text{ m/s}
\]

**STRATEGY:**
The problem involves two separate questions--1) finding the heat transfer coefficient and 2) finding time to freeze. Formulas for heat transfer coefficient are available for flow over slab, cylinder and sphere so we use the ones for slab. The heat transfer coefficient is needed to calculate freezing time since rate of heat transfer from surface definitely influences the rate of freezing (blowing air would freeze it faster than still air). The derivation in the text already includes this \( h \) in the freezing time formula for a slab.

**ASSUMPTIONS:**
Cylinder thickness is small compared to the surface area, so cooling from the edge is ignored, i.e., we are treating this as heat transfer (1D) in a slab.

**SOLUTION:**
1) Calculate the heat transfer coefficient
Since flow is over the side of the patty, which is a flat surface, the formula to use is that for a flat plate. One difficulty is in deciding the length \( L \) of the plate along the flow – \( L = 0 \) at the two ends and \( L = D \) through the centerline. We assume \( L = D \). To choose a formula, we need to know Re, also we are interested in average \( h \), so we would use the equations for average, \( \text{Nu}_L \)
Problem 7.9.7

\[ \text{Re}_L = \frac{u_w D \rho}{\mu} = \frac{0.5\frac{\text{m}}{\text{s}}}{} 0.08[\text{m}] \left( 1.3947 \frac{\text{kg}}{\text{m}^3} \right) = 3.5 \times 10^3 \]  
\[ < 2 \times 10^5 \text{ therefore Laminar flow} \]

The relevant equation is that for flow over a flat plate

\[ \frac{h_L L}{k} = 0.664 \text{ Pr}^{1/3} \text{ Re}^{1/2} \]

\[ h_L = \frac{k}{D} 0.664 \text{ Pr}^{1/3} \text{ Re}^{1/2} \]

\[ = \frac{0.0223}{0.08} \times 0.664 \times 0.72^{1/3} \left( 3.5 \times 10^3 \right)^{1/2} \]

\[ = 9.81 \text{ W/m}^2 \text{K} \]

2) Time to freeze the patty would be calculated by treating the patty as a slab since the thickness dimension is much smaller than the radial dimension. Note also that we are only freezing the water in the meat, not the rest of the solid matrix (proteins, etc.), so the latent heat needs to be multiplied by the fraction of water, 0.75

\[ L = \frac{\text{thickness}}{2} = \frac{0.02 \text{m}}{2} = 0.01 \text{m} \text{ because cooling happens at both sides of the patty.} \]

\[ t_F = \frac{2\rho M}{T_m - T_w} \left[ \frac{L^2}{2k} + \frac{L}{h} \right] \]

\[ = \frac{335 \times 10^3 \left[ \frac{1}{\text{kg}} \right] \times 900 \left[ \frac{\text{kg}}{\text{m}^3} \right] \times 0.75}{-5 - \left( -50 \right)[\text{K}]} \left[ \frac{10^{-4}[\text{m}^2]}{2 \times 1.4[\text{W/mK}]} + \frac{10^{-2}[\text{m}]}{9.81[\text{W/m}^2\text{K}]} \right] \]

\[ = \frac{335 \times 10^3 \times 900 \times 0.75}{-5 \left( -50 \right)} \left[ 3.57 \times 10^{-5} + 1.02 \times 10^{-3} \right] \]

\[ = 5303 \text{ s} \]
\[ = 1.47 \text{ hour} \]

COMMENTS:

By considering freezing time for a slab, here we have ignored cooling effects from the edge.