Problem 8.12.3

KNOWN:
Temperature, blackbody radiation, wavelength range

FIND:
Fraction of energy in the wavelength range (need area between the lines in figure below.

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![Diagram of blackbody radiation with wavelengths 0.35 µm to 0.7 µm and temperature 5800K]

STRATEGY:
As long it is blackbody radiation, for any temperature and wavelength combination, area under the curve is tabulated in Table 8.2 in text. We simply need to use this table for the given temperature and wavelengths.

ASSUMPTIONS:
Solar radiation can be considered blackbody radiation.

SOLUTION:

\[ F_{5800\text{K}}(0.7) - F_{5800\text{K}}(0.35) \]
\[ = F_{0-5800}(0.7) - F_{0-5800}(0.35) = F_{0-4060} - F_{0-2030} \]
\[ = F_{0-4060} - F_{0-2030} \]

From Table 8.2

\[ F_{0-4000} = 0.481246 \]
\[ F_{0-4200} = 0.51636 \]

\[ F_{0-4060} = \frac{4060 - 4000}{4200 - 4000} (0.51636 - 0.481246) + 0.481246 \text{ (linear interpolation)} \]
\[ = 0.49178 \]

Similarly
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\[ F_{0-2030} = \frac{2030 - 2000}{2200 - 2000} (0.10126 - 0.0670396) + 0.0670396 \]
\[ = 0.072173 \]

The fraction is \( F_{0-4060} - F_{0-2030} \)
\[ = 0.49178 - 0.072173 \]
\[ = 0.419608 \]

COMMENTS:

Outside the atmosphere, it is the same fraction. However, on the earth surface, it will not be the same fraction since radiation at different wavelength is absorbed differently by the atmosphere (see figure in text).
Problem 8.12.15

KNOWN:
Needed radiative heat flux

FIND:
The distance between the source of radiation (emitter) and the receiver.

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STRATEGY:
This is obviously a radiative exchange problem. We have a formula in the text for net radiative heat exchange (flux), in terms of surface temperatures and view factor. This net exchange is given to be 200, together with the two surface temperatures, and we need to find the distance parameter that leads to the right view factor.

ASSUMPTIONS:
Geometries are highly highly simplified to be able to use the shape factors.

SOLUTION:
1) Radiative heat flux $= \sigma T^4$

$$= 5.67 \times 10^{-8} \times (273 + 35)^4$$

$$= 510.25 \frac{W}{m^2}$$

2) Net radiative flux inside the infant body $= 200 \frac{W}{m^2}$

Radiative heat flux from infant body $= 510.25 \frac{W}{m^2}$
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Let $A_1$ and $A_2$ be the area of the plates respectively.

$$q_{1-2}'' = 200 \frac{W}{m^2}$$

So, $$200 \frac{W}{m^2} \times A_2 = F_{12} \sigma T_1^4 A_1 - F_{21} \sigma T_2^4 A_2 = A_2 F_{21} \sigma (T_1^4 - T_2^4)$$

$$200 \frac{A_2}{A_1} = F_{21} \sigma (T_1^4 - T_2^4)$$

$$= \left[ \frac{(W_2 + W_1)^2 + 4}{2W_2} \right]^{1/2} - \left[ \frac{(W_2 - W_1)^2 + 4}{2W_2} \right]^{1/2} \sigma (T_1^4 - T_2^4)$$

$$= \left[ \frac{(0.15 + 0.30)^2}{L} + 4 \right]^{1/2} - \left[ \frac{(0.15 - 0.30)^2}{L} + 4 \right]^{1/2} \left(5.67 \times 10^{-8}\right) \left(363^4 - 308^4\right)$$

$$200 = \frac{(0.45)^2 + 4L^2}{0.3} \left[ \frac{(0.15)^2 + 4L^2}{L} \right]^{1/2} \times \left(5.67 \times 10^{-8}\right) \left(363^4 - 308^4\right)$$

(If you solve the equation L = 0.315m = 31.5cm.)

3) Convective heat flux from the body

$$= h(T_s - T_w)$$

$$= 5 \times (35 - 23) = 60 \frac{W}{m^2}$$

4) Net Heat Flux = (Net due to radiation) – (Flux due to convection)

$$= 200 \frac{W}{m^2} - 60 \frac{W}{m^2} = 140 \frac{W}{m^2}$$

5) Evaporation

COMMENTS:
Problem 8.12.9

KNOWN:
Parameters needed to calculate convective and radiative heat transfer from the leaf

FIND:
Steady state leaf temperature

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STRATEGY:
The steady state temperature of the leaf is a result of convective and radiative exchange (loss or gain) of heat. It is assumed that the entire leaf is at one temperature so we do not need to worry about heat conduction inside the leaf, i.e., no conduction equation is needed. Thus, an overall energy balance (at steady-state) would have the only unknown as temperature that we can solve for.

ASSUMPTIONS: 
\( T_{\text{air}} = 278 \, \text{K} \)
\( T_{\text{sky}} = 140 \, \text{K} \)
\( T_{\text{ground}} = 280 \, \text{K} \)

SOLUTION:

\[
\sigma T_{\text{sky}}^4 + \sigma T_{\text{ground}}^4 - 2\sigma T^4 + 2hA(T_{\text{air}} - T) = 0
\]

\[
5.67 \times 10^{-8} \left( 140^4 + 280^4 - 2T^4 \right) = 2 \times 25 \times (T - 278)
\]

\[
7.41 - 2.27 \times 10^{-9} T^4 = T - 278
\]

Iterate after rewriting the above expression as

\[
T = 285.41 - 2.27 \times 10^{-9} T^4
\]

Assume \( T = 278 \). Plugging this on the right side provides \( T = 271.86 \). Repeating this procedure (plugging \( T = 271.86 \) next on the right and so on) leads to \( T = 272.83 \, \text{K} \). Thus temperature of the leaf is \(-0.17^\circ\text{C}\).

COMMENTS:
Higher heat transfer coefficient brings the leaf closer to air temperature, thus warming it. A solution can be to blow air using large fans, which is practiced some places in Florida.