Problem 9.8.27

KNOWN: Henry’s law constant = 304 Pa - m³/mol
Mass transfer coefficient = 0.03 × 10⁻² m/s
Area of floor $A_f = 50 \times 25m$
Volume of room $V_r = 50 \times 25 \times 12m$
Amount of chloroform = 10μg/l

FIND: Source term ($r_a$), air flow rate

SCHEMATIC:

\[ \text{In} \rightarrow \text{Generation} \uparrow h_m (c_s - c) \rightarrow \text{Out} \]

SOLUTION:

Molecular weight of chloroform CHCl₃ = 119.5
Temperature = 20°C = 293K
R = 8.314 J/mol-K

\[ x = \frac{10\mu g}{L} \times \frac{1000 L}{1 m^3} \times \frac{1 g}{10^6 \mu g} = \frac{1 mol}{119.5 g} \]

\[ = \frac{10^{-2}}{119.5} \text{mol/m}^3 \]

Partial pressure = $p = H_o x = 304 \frac{Pa - m^3}{mol} \times \frac{10 \times 10^{-3}}{119.5} \frac{mol}{m^3}$

Concentration at surface ($c_s$) = \[ \frac{pM}{RT} \]

Volumetric source term $r_a = h_m \Delta c \frac{kg}{V_r}$

\[ = 0.03 \times 10^{-2} (0.001248 - c) \frac{kg}{V_r} \]

Perform mass balance of chloroform in the room air:
where $\dot{v}$ = volume flow rate
$c = \text{concentration of CHCl}_3 \text{in room air, kg/m}^3$
$c_s = \text{concentration of CHCl}_3 \text{in room air at air water interface, kg/m}^3$
$V_r = \text{volume of room}$

Problem 9.8.27

In - Out + Generation = change in storage
\[
\left[ 0 \cdot \dot{v} - c \cdot \dot{v} + h_m \left( \frac{\dot{v}}{V_r} \right) V_r (c_s - c) \right] \Delta t = V_r \cdot \Delta c
\]

\[
\frac{\Delta c}{\Delta t} = \frac{h_m A c_s - (\dot{v} + h_m A) c}{V_r}
\]

\[
\frac{dc}{dt} = -\left( \frac{\dot{v} + h_m A}{V_r} \right) c + \frac{h_m A c_s}{V_r}
\]

\[
\frac{dc}{dt} = -bc + a = a - bc
\]

Where \( b = \frac{\dot{v} + h_m A}{V_r} \) and \( a = \frac{h_m A c_s}{V_r} \)

\[
\int_{c_i}^{c} \frac{dc}{a - bc} = \int_{t_i}^{t} dt
\]

\[
-\frac{1}{b} \ln|a - bc|_{c_i}^{c} = t
\]

\[
\ln \left( \frac{a - bc}{a - bc_i} \right) = -bt
\]

Concentration, \( c = \frac{a}{b} - \frac{(a - bc_i) e^{-bt}}{b} \)

For steady state, \( t \to \infty \)

\[
c = \frac{a}{b} = \frac{h_m A c_s}{V_r} \times \frac{V_r}{\dot{v} + h_m A} = 0.375 \mu g/l = 0.375 \times 10^{-3} \text{ g/m}^3
\]

\[
\frac{h_m A c_s}{\dot{v} + h_m A} = c
\]

\[
\frac{0.03 \times 10^{-2} \times 50 \times 25 \times .001248}{\dot{v} + (0.03 \times 10^{-2} \times 50 \times 25)} = 0.375 \times 10^{-3}
\]

Volume flow rate \( \dot{v} = 0.873 \text{ m}^3/\text{s} \)

Plot of concentration vs. time:

**Problem 9.8.27**
**Problem 9.8.5**

**KNOWN:**
Oxygen content of air in contact with blood

**FIND:**
1. Concentration of dissolved oxygen is the blood plasma
2. Does the concentration seem reasonable?
3. What other factors are involved in the transport of oxygen to the cells?

**SCHEMATIC AND GIVEN DATA**

![Schematic showing capillary beds and alveoli]

- $x_{O_2,\text{air}} = 14.5\%$ (mole fraction)
- $H = 5.21 \times 10^4 \text{ atm/mole fraction of } O_2 \text{ in air}$
- $M_{O_2} = 32\text{ g}$
- $M_{H_2O} = 18\text{ g}$

**STRATEGY:**
Assuming air and blood (treated same as water here) are in contact, oxygen in blood (a liquid) and oxygen in air (a gas) are in equilibrium with each other. The amount of oxygen in blood is related to the amount of oxygen in air, according to Henry's law.

**ASSUMPTIONS:**
1. Blood plasma has the same material properties as water.
2. The concentration of oxygen in the inhaled air is not affected by the transport of oxygen into the blood plasma.
3. The blood plasma is well-mixed so the concentration of dissolved $O_2$ is constant throughout the vessel.
Problem 9.8.5

SOLUTION:

1) $p_{O_2,\text{air}} = (\text{mole fraction of } O_2,\text{air})(P_{\text{air}})$
   
   $= (0.145)(1 \text{ atm})$
   
   $= 0.145 \text{ atm}$

2) Since the air is in equilibrium with the blood vessel, the $p_{O_2,\text{air}} = p_{O_2,\text{vessel}}$

   $p_{O_2} = H c_{O_2}$ where $c_{O_2} = \text{concentration of } O_2$

   $c_{O_2} = \frac{p_{O_2}}{H} = \frac{0.145 \text{ atm}}{5.21 \times 10^4 \text{ atm/mole fraction } O_2} = 2.783 \times 10^{-6} \text{ mol } O_2/\text{total mol}$

   $\approx 2.783 \times 10^{-6} \text{ mol } O_2 / \text{ mol } H_2O \text{ (mole fraction of } O_2 << 1)$

   $= 2.783 \times 10^{-6} \frac{\text{mol } O_2}{\text{mol } H_2O} \times \frac{1 \text{ mol } H_2O}{18 \text{ g } H_2O} \times \frac{32 \text{ g } O_2}{1 \text{ mol } O_2} = 4.95 \times 10^{-6} \text{ g } O_2/\text{g } H_2O$

   $c_{O_2} = 4.95 \times 10^{-4}\% \text{ (by mass)}$

The value for $c_{O_2}$ is an extremely low concentration of dissolved $O_2$ in blood, so another method of $O_2$ transportation (i.e. selective binding to hemoglobin) must be employed to efficiently transport $O_2$ in the body.

3) Several (diffusive) resistances are involved in the transport of oxygen to the cells from the blood plasma.

COMMENTS:
Problem 9.8.12

KNOWN:
Relation between moisture content of air and moisture content of wood at equilibrium, when the air is in contact with wood.

FIND:
Amount of moisture gained by wood when moisture in the air increases.

SCHEMATIC AND GIVEN DATA:

STRATEGY:
The equilibrium relationship between moisture in wood and air, given in Figure 9.8, can be used to find moisture in wood at two moisture levels in air (two relative humidity values).

ASSUMPTIONS:
Sufficient contact time is present between wood and air so the entire wood is in equilibrium with air.

SOLUTION: Change in Moisture Content of Wood
From Figure 9.8 in text on equilibrium moisture for wood
At 20% RH, moisture content is \( \frac{0.04 \text{ kg water}}{\text{kg wet wood}} \)
At 80% RH, moisture content is \( \frac{0.17 \text{ kg water}}{\text{kg wet wood}} \)
Since the wet wood (water + dry wood) is different in the two situations, the two moistures cannot be subtracted directly from each other.

Consider
\[
\frac{\text{kg water}}{\text{kg dry wood}} = \frac{\text{kg water}}{\text{kg wet wood - kg water}}
\]
\[
= \frac{1}{\frac{\text{kg of wet wood}}{\text{kg of water}}} - 1
\]

For \( \frac{\text{kg of water}}{\text{kg of wet wood}} = 0.17, \)
\[
\frac{\text{kg of water}}{\text{kg of dry wood}} = \frac{1}{\frac{0.17}{0.17} - 1}
\]
\[
= \frac{0.17}{0.83}
\]
\[
= 0.205
\]
Likewise, .04 kg water/kg wet wood = .042 kg water/kg dry wood

Moisture gained
\[
\frac{.205 \text{ kg water}}{\text{kg dry wood}} - \frac{.042 \text{ kg water}}{\text{kg dry wood}} = \frac{.163 \text{ kg water}}{\text{kg dry wood}}
\]

COMMENTS:
Problem 9.8.16

KNOWN:
Desired final bacterial concentration can be reached by heating at 121°C for 6 minutes.

FIND:
How many minutes to heat to reach the same final bacterial concentration at a lower temperature of 111°C

SCHEMATIC AND GIVEN DATA

STRATEGY:
We have the equation for concentration as function of time for a first order reaction, at a given temperature. If the final concentration is kept constant, temperature and time are related. Thus, for a different temperature, we can find the time.

ASSUMPTIONS:
First order kinetics for destruction is valid over the temperature range of interest.

SOLUTION:
\[
c/c_0 = e^{-k_0 e^{E/RT_1} t_1} = e^{-k_0 e^{E/RT_2} t_2}
\]
c/c_0 needs to stay the same at temperatures T_1 & T_2

\[
e^{E/RT_1 t_1} = e^{E/RT_2 t_2}
\]

\[
e^{E/RT_1} t_1 = e^{E/RT_2} t_2
\]

\[
t_2 = e^{-E/RT_2} t_1
\]

\[
t_2 = e^{-3575 [1/394 1/384]} \text{ min}
\]

\[
t_2 = (10.62)(6) \text{ min}
\]

\[
t_2 = 63.74 \text{ min}
\]

COMMENTS: