**Problem 2.7.4**

**KNOWN:** Average surface temperature of sun.

**FIND:** Determine the energy flux at the surface of the sun.

**SCHEMATIC AND GIVEN DATA**

![Sun with label Ts=5800K](sun.png)

**STRATEGY:**

**ASSUMPTIONS:**

1. Sun is a perfect black body. \( \varepsilon = 1. \)
2. Average surface temperature = 5800 K.

**SOLUTION:**

\[ q'' = \sigma \varepsilon T^4 \]

\[ = 5.67 \times 10^{-8} \frac{W}{m^2K^4}(1)(5800K)^4 \]

\[ = 6.416 \times 10^7 \frac{W}{m^2} \]

**COMMENTS:**

1. The energy flux is greater than the given flux of 1353 W/m² by 4 orders of magnitude. The decrease from sun surface to earth surface is due to the fact that the amount of energy leaving solar surface is going in all directions and only a fraction of this is intercepted by earth, due to large distance between the two and the relative sizes of the sun and the earth.
**Problem 2.7.6**

**KNOWN:**

\[ T_{air} = 20^\circ C \]
\[ T_{canary} = 33^\circ C \]
\[ H_{canary} = 25.2 \text{ W/m}^2\text{K} \]
\[ \Delta T \text{ (temp. diff. between exhaled and inhaled)} = 4.3^\circ C \]

Ventilation rate = 0.74 cc/s
\[ c_{p,\text{air}} = 1.0066 \text{ kJ/kg} \cdot \text{K} \]
\[ \rho_{\text{air}} = 1.16 \text{ kg/m}^3 \]

Canary can be modeled as a cylinder with a diameter of 7cm and height of 9cm

Heat gained by radiation from surrounding = 11.5 W

**FIND:**

1) Amount of heat lost by radiation

\[ q = A\sigma T^4 \]
\[ A = 2 \left( \pi \left( \frac{d}{2} \right)^2 \right) + \pi dh \]
\[ = 2 \left( \pi \left( \frac{0.07m}{2} \right)^2 \right) + \pi (0.07m)(0.09m) \]
\[ = 0.0275 \text{ m}^2 \]

\[ q = (0.0275m^2) \left( 5.67 \times 10^{-8} \frac{W}{m^2 K^4} \right) (273.15K + 33K)^4 \]
\[ = 13.7 \text{ W} \quad \text{gross heat lost} \]

Net heat lost = 13.7W - 11.5W = 2.2 W

2) Find amount of heat transferred by convection to surrounding air

\[ Q = h \ A \ (T_{canary} - T_{air}) \]
3) Find amount of heat transferred in the exhaled air

\[ q = \dot{m} c_p \Delta T \]

\[ = \dot{v} \rho c_v \Delta T \]

\[ = (0.74 \text{ cc/s}) \left( \frac{1 \text{ m}^3}{1 \times 10^6 \text{ cc}} \right) \left( 1.16 \frac{\text{ kg}}{\text{ m}^3} \right) \left( 1.0066 \frac{\text{ kJ}}{\text{ kg} \cdot \text{ K}} \right)(4.3^\circ \text{C}) \]

\[ = 3.72 \times 10^{-3} \frac{\text{ J}}{\text{ s}} \]

\[ = 3.72 \text{ mW} \]

4) Total power = 2.2 W + 9.01 W + 3.72 \times 10^{-3} W = 11.21 W
Problem 4.8.26

KNOWN:

Termite mound covering a large area on the ground, with air blowing over its top surface, maintaining the surface at a constant temperature. Temperatures of the top and bottom surfaces, and thermal conductivity of the mound are known.

FIND:

(1) Governing equation and boundary conditions for the problem.

(2) Temperature as a function of vertical position from the ground.

(3) Location above the ground where the temperature is maximum in terms of other parameters.

(4) Reasons for why the location of maximum temperature is not at mid-height.

(5) Equations from which you can calculate numerical values for the amount of heat that the termites are producing and location of maximum temperature, given that the maximum temperature is 30°C.

SCHEMATIC AND GIVEN DATA

Temperature of top surface, $T_2 = 27°C$

Temperature of bottom surface, $T_1 = 29°C$
Problem 4.8.26

Thermal conductivity of the mound, \( k = 0.4 \text{ W/mK} \)

**STRATEGY:**
The area of the mound being large, we can approximate the process as 1D, with heat generation. The temperatures at the two ends not being equal, we cannot use the solution for heat generation in a slab developed in the textbook. Thus, we have to start from the governing equation.

**ASSUMPTIONS:**
The mound can be approximated to a slab with internal heat generation. Steady state has been reached.

**SOLUTION:**

1. **Governing equation**

   \[
   0 = k \frac{\partial^2 T}{\partial x^2} + Q
   \]

   \[
   \frac{\partial^2 T}{\partial x^2} = -\frac{Q}{k}
   \]

   **Boundary conditions**

   \( x = 0, T = T_1 \)

   \( x = L, T = T_2 \)

2. **Integrating the governing equation,**

   \[
   \frac{dT}{dx} = -\frac{Q}{k} x + C_1
   \]

   Integrating again,

   \[
   T = -\frac{Q}{2k} x^2 + C_1 x + C_2
   \]

   Using the first boundary condition,
Problem 4.8.33

Known:
Steady state heat transfer in concentric spheres. Two conductive layers and convective transport from ambient.

Find:
Temperature for a snow and cardboard model and for a U-Vacua model only. Find all three heat flows. The overall thermal resistance and the heat flow from snow to ambient. Percent of snow that melted.

Schematic and Given Data:
k_c = 0.067 W/mK
k_U = 0.00070 W/mK
R_i = 0.5 m
R_m = 0.6 m
R_o = 0.62 m

Assumptions:
Steady State. Snow is at constant temperature.

Strategy:
Going down the decision tree, this is transient heat transfer in a spherical shell. Since the problem is asking to derive the equation for temperature starting from governing equation, we would proceed to do so as opposed to writing the solution if it is already available. Start with the heat equation and cancel transient, convective, and heat generation term. Apply boundary conditions and solve for temperature. Then, take the derivative of temperature to obtain heat flow. Rearrange heat flow in terms of temperature difference and resistance, to find the resistance quantity. Heat flow from the snow to air will be obtained by using the total thermal resistance of the cardboard, insulation and air and the total temperature difference. Calculate the heat gained over 40 hours using heat flow. Calculate mass of snow and find percent that would melt.

Solution:

1) \[
\rho C_p \left( \frac{\partial T}{\partial t} + \nabla \cdot T \right) = k \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right] + Q
\]

\[
\frac{\partial T}{\partial t} = 0 \quad \text{steady state}
\]

\[
\nabla \cdot T = 0 \quad \text{no convection}
\]

\[
\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} = 0 \quad \text{only radial variance}
\]

\[
Q = 0 \quad \text{no generation}
\]
Conductive flow is defined as:

\[ 0 = k \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) \right] \]

\[ T(r = R_i) = T_i \]
\[ T(r = R_m) = \square \]

2) \[ 0 = k \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) \right] \]

\[ 0 = \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) \]

\[ c_1 = r^2 \frac{\partial T}{\partial r} \Rightarrow c_1 = \frac{\partial T}{r^2} \]

\[ T = c_2 - \frac{c_1}{r} \]

\[ T_i = c_2 - \frac{c_1}{R_i} \]

\[ T_m = c_2 - \frac{c_1}{R_m} \]

\[ T_m - T_i = c_1 \left[ \frac{1}{R_i} - \frac{1}{R_m} \right] \]

\[ c_1 = (T_m - T_i) \left( \frac{R_m R_i}{R_m - R_i} \right) \]

\[ R_i T_i = R_i c_2 - c_1 \]

\[ R_m T_m = R_m c_2 - c_1 \]

\[ c_2 = \frac{R_i T_i - R_m T_m}{R_m - R_i} \]

\[ T = \frac{R_i T_i - R_m T_m}{R_m - R_i} - \frac{(T_m - T_i)}{r} \left( \frac{R_m R_i}{R_m - R_i} \right) \]

3) Writing the heat flow in terms of temperature difference and resistance,

\[ q_r = -k A \frac{dT}{dr} \]

\[ q_r = -k A \frac{dT}{dr} = -k \left( 4\pi r^2 \right) \left[ \frac{(T_m - T_i)}{r^2} \left( \frac{R_m R_i}{R_m - R_i} \right) \right] = -4\pi k \left( T_m - T_i \right) \left( \frac{R_m R_i}{R_m - R_i} \right) \]
Problem 5.9.15

KNOWN: A donor heart is packed in ice and saline for travel to a recipient.

FIND: 1) The temperature at the center of the heart after 3 hrs with radial conduction.
       2) Compare the result of 1) to the result assuming axial & radial conduction.

SCHEMATIC AND GIVEN DATA

\[ \alpha = 1.4 \times 10^{-7} \text{ m}^2/\text{s} \]
\[ T_i = \text{initial temp of heart} = 37^\circ \text{C} \]
\[ T_s = \text{temp of saline solution} = 0^\circ \text{C} \]
\[ R = \text{radius of cylinder approximation of the heart} = 3.75 \text{ cm} \]
\[ h = \text{height of cylinder approx of the heart} = 9 \text{ cm} \]

STRATEGY:

ASSUMPTIONS:

1) The heart is a solid finite cylinder with constant, equal thermal properties in all directions.
2) The time the heart is between bodies and in ice, there is negligible metabolic heat production.
3) The saline solution is agitated so that it maintains the surface temperature of the heart at the constant value of the saline solution temperature.

SOLUTION:

1) Assuming negligible axial heat conduction, therefore infinite cylinder.

\[ n = r / R = 0 \text{ (for center of the heart) } \]
\[ m = k / hR = 0 \text{ (since surface temperature is constant, } h \to \infty ) \]
\[ F_0 = \frac{\alpha t}{R^2} = \frac{(1.4 \times 10^{-7}) \times (3) \times (3600)}{(3.75 \times 10^{-2})^2} = 1.075 \]
\[ F_0 > 0.2, \text{ so Heisler Charts can be used} \]

B.C : 1) dT/dr at r=0 will be = 0 for t>0 (By Radial Symmetry)
      2) T at radius .0375m = 0 degree Celcius for t>0
     I. C : 1) T at t=0 is 37 degree celcius for 0<r<R
      2) Can not ignore the axial conduction any more, Hence
      GE : dT/dt = alpha x (1/r x d/dr(rdT/dt) + d2T/dz2)

B.C : same as part one for r
For z : T at z=0 is 0 degree C for t>0
       T at z=0.09m is 0 degree C for t>0
      IC: would be the same