Problem 6.10.12

Can use the properties at the closest tabulated temperature (Table C.12 in text) of 280 K, to avoid any interpolation (which would be the right way to do).

**ASSUMPTIONS:** Use the film temperature as an average of T_surface at 7 degC and the T_infinity at 5 degC

**SOLUTION:**

Use Pg. 111, eqn. 6.52, for forced convection over a sphere, to find h.

\[
Re_D = \frac{u_D D}{\mu} = \frac{0.04 \text{ m/s}}{1.422 \times 10^{-3} \text{ kg/m/s}} \times \left( \frac{0.12 \text{ m}}{999.9 \text{ kg/m}^3} \right) = 3375; \quad \text{Thus, } 3.5 < Re_D < 7.6 \times 10^4
\]

\[
Pr = 10.26, \quad \text{Thus, } 0.71 < Pr < 380.
\]

Using Eqn. 6.52,

\[
Nu_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4}
\]

\[
= 2 + \left[ (0.4)(3375)^{1/2} + (0.06)(3375)^{2/3} \right](10.26)^{0.4}
\]

\[
= 95.2
\]

\[
Nu_D = \frac{h D}{k}
\]

\[
h = \frac{k Nu_D D}{(95.2)} = \frac{0.582 \frac{W}{m \cdot K}}{(0.12 \text{ m})} = 461.9 \frac{W}{m^2 \cdot K}
\]

\[
Bi = \frac{h R}{k_{apple}} \text{ from Pg. 73}
\]

\[
= \frac{\left( 461.9 \frac{W}{m^2 \cdot K} \right)(0.060 \text{ m})}{\left( 0.418 \frac{W}{m \cdot K} \right)} = 66.3
\]

**\( Bi = 66.3 \gg 0.1 \), so a lumped parameter approach will not work. Using Heisler chart on P. 329, \( n = 0 \) since we want the center temp**
\[ m = \frac{1}{Bi} = \frac{1}{66.3} = 0.015 \approx 0 \]

\[ \frac{T - T_e}{T_i - T_w} = \frac{8 - 5}{25 - 5} = 0.15 \]

From the chart, \( Fo \approx 0.30 \)

\[ Fo = \frac{at}{R^2}, \text{ so } t = \frac{Fo R^2}{\alpha} \]

\[ \alpha = \frac{k}{\rho c_p} = \frac{0.418 \frac{W}{m \cdot K}}{740 \frac{kg}{m^3} \left(3600 \frac{J}{kg \cdot K}\right)} = 1.57 \times 10^{-7} \frac{m^2}{s} \]

\[ t = \frac{(0.30)(0.060 m)^2}{1.57 \times 10^{-7} \frac{m^2}{s}} \]

\[ t = 6879 \text{ s} = 115 \text{ min} \]

**COMMENTS:**
Problem 6.10.13

KNOWN: A cattle watering fountain has a free water surface with length width 0.30 m, with the length parallel to the prevailing wind.

FIND: Find the power rating needed for an electrical heating element that will keep the well-mixed water at 10° C when the wind speed is 1.0 m/s and the air temperature is -23° C.

SCHEMATIC AND GIVEN DATA

STRATEGY:

ASSUMPTIONS: Assume: 1) the bottom and sides of the water container are perfectly insulated, 2) the water container is full of water, so that edge or boundary effects are negligible, and 3) you can neglect evaporative cooling.

SOLUTION: Use forced convection over a flat plate to find h, then use \( q = h A (T - T_\infty) \) to find \( q \).

Use p. 89, forced convection over a flat plate, eqn. 6.33 or 6.35.

\[
T_{\text{film}} = \frac{10 + (-23)}{2} = 6.5^\circ C = 266.5 K \approx 270 K
\]

\[
Re = \frac{u_\infty L \rho}{\mu} = \left( \frac{1.0 \text{ m}}{s} \right) \left( 0.60 \text{ m} \right) \left( 1.3082 \frac{\text{kg}}{\text{m}^3} \right) \left( 1.7005 \times 10^5 \frac{\text{kg}}{\text{m} \cdot \text{s}} \right) = 46158 \text{ which is} < 2 \times 10^5, \text{ so flow is laminar.}
\]

Therefore, use eqn. 6.35.
\[ \text{Nu}_L = 0.664 \Pr^{1/3} \text{Re}^{1/2} = \frac{hL}{k} \]

\[ \text{Nu}_L = 0.664 \left(0.716\right)^{1/3} \left(53025\right)^{1/2} \]

\[ \text{Nu}_L = 127.6 \]

\[ h = \frac{\text{Nu}_L k}{L} = \frac{127.6}{0.6} \left(\frac{0.02388 \ W}{m \cdot K}\right) = 5.079 \ W \ m^2 \cdot K \]

\[ q = h A (T-T_e) \]

\[ q = \left(5.079 \ W \ m^{-2} \cdot K\right) \left[\left(0.60 \ m\right) \left(0.30 \ m\right)\right] \left(10^\circ \text{C} - (-23^\circ \text{C})\right) \]

\[ q = 30.2 \ W \]

**COMMENTS:**
1. Diameter of a pea
2. Fluid properties

The smallest possible value of heat transfer coefficient

1. Natural connection over sphere
2. Pea as a sphere

1. Heat transfer coefficient is smallest when the relative velocity between the fluid and the particle is zero

→ Flow over sphere. Forced convection

\[
\frac{hD}{k_{fluid}} \geq 2 \quad h \geq \frac{2-k_{fluid}}{D} = \frac{(2)(0.5)}{0.005} = 200 \text{ W/m}^2\text{K}
\]

2. When you decide the heating time based on the worst case scenario as shown above, most parts of food will be overheated and thus overcooked, losing nutrients.
Problem 6.10.20

KNOWN:

FIND:

SCHEMATIC AND GIVEN DATA

STRATEGY:

ASSUMPTIONS:

SOLUTION:

(1) Heat lost by forced convection (blowing wind) is more than natural convection (still air).

(2) First we calculate the Reynold’s number to check if the flow is laminar or turbulent

\[ T_{\text{film}} = \frac{20 + 10}{2} = 15^\circ \text{C} = 288 \text{K} \approx 290 \text{K} \]

\[ \text{Re}_L = \frac{uL}{v} = \frac{8.96 \times 0.07 \times 1.2177}{1.7985 \times 10^{-5}} = 4.25 \times 10^4 \]

So, the flow over flat plate is laminar. The formula for average Nusselt’s Number over the length of the cheek surface is

\[ \text{Nu} = 0.664 \times \left( \text{Re}_L \right)^{\frac{1}{2}} \left( \text{Pr} \right)^{\frac{1}{3}} \]

\[ = 0.664 \times \left( 4.25 \times 10^4 \right)^{\frac{1}{2}} \left( 0.710 \right)^{\frac{1}{3}} = 122.07 \]

Since \( \text{Nu}_L = \frac{hL}{k} \)

\[ h = \frac{\text{Nu}_L \times k}{L} = \frac{122.07 \times 0.02547}{0.07} = 44.42 \text{ W m}^{-2} \text{K}^{-1} \]

(3) Average heat flux over the cheek length

\[ = h_L \left( T_s - T_\infty \right) \]

\[ = 44.42 \text{ W m}^{-2} \text{K}^{-1} \times (20 - 10) \text{K} \]

\[ = 444.2 \text{ W m}^{-2} \]
Problem 6.10.20

(4) Natural convection is there because heat transfer near the surface is still taking place as there is movement of air due to change in density.

(5) Heat flux calculated in Part (3)

\[ \frac{W}{m^2} = h_{NC} \times (T_s - T_\infty) \]

Since, \( h_{NC} = 10 \frac{W}{m^2 \text{k}} \)

Therefore, \( T_\infty = 20 - \left( \frac{444.2}{10} \right) = -24.42^\circ \text{C} \)

(6) The lowest temperature is attained where \( h \to \infty \) and the surface temperature is equal to air temperature. Thus, minimum possible temperature = \( 10^\circ \text{C} \).

COMMENTS: